

Radiative decay of the lightest neutralino in an R -parity violating supersymmetric theory

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In an R -parity violating supersymmetric scenario, the lightest neutralino $\tilde{\chi}_1^0$ is no longer a stable particle. We calculate the branching ratio for the decay mode $\tilde{\chi}_1^0 \rightarrow \nu \gamma$ which occurs at the one-loop level. Taking into account bilinear as well as trilinear lepton number violating interactions as the sources of R -parity violation, we make a detailed scan of the parameter space, both with and without gaugino mass unification and including the constraints on the neutrino sector from the recent SuperKamiokande results. This study enables one to suggest interesting experimental signals distinguishing between the two types of R -parity breaking, and also to ascertain whether such radiative decays can give rise to collider signals of the type $\gamma\gamma + \cancel{E}$ from pair-produced neutralinos. [S0556-2821(99)07521-9]

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I. INTRODUCTION

The decay of a heavy particle with a photon in the final state can often lead to experimental signals for physics beyond the standard model. The recent literature contains many studies, both experimental and theoretical, devoted to signals of the type $\gamma\gamma + \cancel{E}$, $\gamma + \cancel{E}$, etc. [1,2], in the context of various new physics scenarios. A rather large fraction of such studies are concerned with supersymmetric (SUSY) [3] theories. For example, in models with gauge mediated supersymmetry breaking (GMSB) [4,5], the decay of the lightest neutralino ($\tilde{\chi}_1^0$) (next-to-lightest supersymmetric particle in these models) into a photon and a gravitino can give rise to $\gamma\gamma + \cancel{E}$ (or $\gamma + \cancel{E}$) which can have substantial event rates even after subtracting the standard model backgrounds. The signal $\gamma\gamma + \cancel{E}$ is in fact the most probable discovery channel of GMSB. Side by side, the radiative decay of the second lightest neutralino ($\tilde{\chi}_2^0$), in a scenario based on $N=1$ supergravity (SUGRA), into a photon and the lightest neutralino ($\tilde{\chi}_1^0$, the lightest supersymmetric particle in this class of models) can sometimes give rise to similar final states in colliders from a pair of $\tilde{\chi}_2^0$'s or from a $\tilde{\chi}_2^0 \tilde{\chi}_1^0$ pair. It requires a careful investigation to distinguish the latter type of signals from the former to establish the reality of GMSB in relation to the SUGRA-type mode of supersymmetry breaking [6].

In view of this, it is important to know whether there can be other new physics options which allow radiative decay of a heavier particle into a photon and another invisible particle, leading to signals of the same kind as those discussed above, and, in some cases, faking the GMSB signals which are being looked for so carefully. It will be obvious that the closest kinematical resemblance to the GMSB signal is borne when the invisible particle is a near-massless neutral fermion, for which the immediate candidate is a neutrino. Now, radiative decay of a heavier neutral fermion into a neutrino becomes a distinct possibility in R -parity violating SUSY [7] where the lightest neutralino can decay in this vein at the one-loop level. This motivates one to undertake an exhaustive study of the radiative decay $\tilde{\chi}_1^0 \rightarrow \nu \gamma$ in an R -parity violating scenario, to see whether in some region of the parameter space this

kind of a decay can have an appreciable branching ratio in spite of the allowed tree-level decay channels of the lightest neutralino. In this paper, we have attempted such a study, taking into account R -parity violation through both bilinear and trilinear terms in the superpotential.

It should be mentioned that $\tilde{\chi}_1^0 \rightarrow \nu \gamma$ has been calculated in some earlier works [8] where either the composition of the lightest neutralino has been confined to certain limits or some specific regions of the SUSY parameter space have been adhered to. Our purpose, on the other hand, is to make a detailed scan of the parameter space, and the results presented by us highlight those particular regions where the branching ratio can be of the largest magnitude. This, we feel, is necessary to establish the authenticity of, for example, GMSB signals during collider searches. We also go beyond the assumption of gaugino mass unification at the scale of grand unified theories (GUT's) and present some results with the $SU(2)$ and $U(1)$ gaugino masses treated as independent parameters, to probe whether radiative neutralino decays with enhanced rates ensue upon freeing the SUSY standard model from a GUT embedding. And last, we take into account the fact that R -parity violating SUSY is being invoked in recent times to explain the generation of neutrino masses [9] in consonance with the SuperKamiokande data on atmospheric neutrinos [10]. Thus a section of our results pertains to that particular region of the parameter space where the indicated hierarchy of neutrino masses, together with large angle mixing between the second and the third generations, is reproduced.

The paper is organized as follows. In Sec. II, we describe the basic framework of the R -parity violating model used in our calculations. In Sec. III, some details of the loop calculation are presented. Section IV contains our numerical results for various scenarios as well as for different choices of the parameters. We summarize and conclude in Sec. V. Detailed forms of the loop integrals are outlined in the Appendix.

II. BASIC FRAMEWORK

The fact that all the particles in the standard model carrying baryon (B) and lepton (L) number are fermions, with

fixed gauge multiplet structures, implies that B and/or L cannot be violated by one unit. Thus one cannot write down renormalizable terms in the Lagrangian which violate B or L or both. This is no longer true in the case of supersymmetric theories where the particle spectrum is now doubled and baryon number and lepton number are assigned to the supermultiplets. One can now have scalar particles containing B or L , and it is possible to have terms with ΔL or $\Delta B = 1$. However, the scenario has to be made consistent with the non-observation of proton decay which requires the simultaneous violation of B and L . This is ensured in an over-restrictive manner in the minimal supersymmetric standard model (MSSM), by imposing a discrete multiplicative symmetry called R parity defined as $R = (-1)^{L+3\bar{B}+2S}$, which equals $+1$ for standard model particles and -1 for the superpartners. An immediate consequence of R -parity conservation is that the lightest supersymmetric particle (LSP) is stable.

The conservation of R parity, however, is not prompted by any strong theoretical reason, and theories where R is violated through nonconservation of *either* B or L are perfectly consistent with stability of the proton. Such scenarios can be studied by generalizing the MSSM superpotential to the following form:

$$W = W_{MSSM} + W_K \quad (1)$$

with

$$W_{MSSM} = \mu \hat{H}_1 \hat{H}_2 + h_{ij}^l \hat{L}_i \hat{H}_1 \hat{E}_j^c + h_{ij}^d \hat{Q}_i \hat{H}_1 \hat{D}_j^c + h_{ij}^u \hat{Q}_i \hat{H}_2 \hat{U}_j^c \quad (2)$$

and

$$W_K = \lambda_{ijk} \hat{L}_i \hat{L}_j \hat{E}_k^c + \lambda'_{ijk} \hat{L}_i \hat{Q}_j \hat{D}_k^c + \lambda''_{ijk} \hat{U}_i \hat{D}_j^c \hat{D}_k^c + \epsilon_i \hat{L}_i \hat{H}_2. \quad (3)$$

In this paper we shall assume that the B -violating piece λ'' is not there. Also, we discuss the L -violating terms in two separate categories for the convenience of analysis, considering, in turn, W_K with either the bilinear ($\epsilon_i L_i H_2$) [11–13] or the trilinear (λ and λ') [14] terms existing in the superpotential at a time.

Case (1): $W_K = \epsilon_3 \hat{L}_3 \hat{H}_2$

We simplify the analysis here by assuming a bilinear R -parity violating term involving only the third leptonic generation. All the important features of such a scenario can be seen within simplified framework.

At first sight it appears that the ϵ term can be rotated away from the superpotential by properly redefining the L_3 and H_1 fields, so that

$$\hat{H}_1' = \frac{\mu \hat{H}_1 + \epsilon_3 \hat{L}_3}{\sqrt{\mu^2 + \epsilon_3^2}} \quad (4)$$

$$\hat{L}_3' = \frac{-\epsilon_3 \hat{H}_1 + \mu \hat{L}_3}{\sqrt{\mu^2 + \epsilon_3^2}}. \quad (5)$$

Now as a result of this rotation the superpotential takes the form

$$W = \mu' \hat{H}_1' \hat{H}_2 + h_{33}^d \hat{L}_3' \hat{H}_1' \hat{E}_3^c + \frac{h_{ij}^d \epsilon}{\mu'} \hat{Q}_i \hat{H}_1' \hat{D}_j^c + h_{ij}^u \hat{Q}_i \hat{H}_2 \hat{U}_j^c - \frac{h_{ij}^d \epsilon}{\mu'} \hat{Q}_i \hat{L}_3' \hat{D}_j^c - \frac{h_{ij}^l \epsilon}{\mu'} \hat{L}_i \hat{L}_3' \hat{E}_j^c + \frac{h_{ij}^l \mu}{\mu'} \hat{L}_i \hat{H}_1' \hat{E}_j^c \quad (6)$$

which means that the λ - and λ' -type terms are generated in general and in the last two terms $i=1,2$.

But more importantly, it should be noted that the scalar potential has terms of the following forms in the original basis:

$$V_{scal} = M_{L_3}^2 \tilde{L}_3^2 + m_1^2 H_1^2 + B_1 \mu H_1 H_2 + B_2 \epsilon \tilde{L}_3 H_2 + \mu \epsilon_3 \tilde{L}_3 H_1 + \dots, \quad (7)$$

where we have written down the F term and soft breaking terms relevant for our purpose here. It is evident that such terms will generate, in general, a vacuum expectation value (VEV) v_3 for the sneutrino. The basis rotation regenerates such terms, and therefore the sneutrino VEV is in general non-vanishing as the special consequence of bilinear R -parity violation whenever soft SUSY breaking is there [15,16].

Thus there are two ways of parametrizing a bilinear R -parity violating scenario: (i) with the ϵ term present in the superpotential together with a VEV of the sneutrino and (ii) with the ϵ term rotated away from the superpotential but the VEV of the sneutrino (in the rotated basis) embodying the L -violating effects that the former would imply in, say, the neutralino and chargino mass matrices. Of course, the parameter ϵ takes refuge in this case in the scalar potential together with the soft breaking parameter B_2 . B_2 can be eliminated by using the conditions for electroweak symmetry breaking, if ϵ and v_3 are used as independent variables. Our calculations here are done in a basis where both ϵ and v_3 (VEV of the sneutrino) are present.

Now, the presence of ϵ and v_3 will induce a kind of mixing in the fermionic as well as in the scalar sector of the theory, which is typical of bilinear R -parity violation. In the fermionic sector neutralinos will mix with the tau neutrino and the charginos with the tau lepton. Consequently, the neutralino mass matrix takes the following form:

$$\mathcal{M}_{\tilde{\chi}_1^0} = \begin{pmatrix} 0 & -\mu & \frac{g v}{\sqrt{2}} & -\frac{g' v}{\sqrt{2}} & -\epsilon \\ -\mu & 0 & -\frac{g v'}{\sqrt{2}} & \frac{g' v'}{\sqrt{2}} & 0 \\ \frac{g v}{\sqrt{2}} & -\frac{g v'}{\sqrt{2}} & M & 0 & -\frac{g v_3}{\sqrt{2}} \\ -\frac{g' v}{\sqrt{2}} & \frac{g' v'}{\sqrt{2}} & 0 & M' & \frac{g' v_3}{\sqrt{2}} \\ -\epsilon & 0 & -\frac{g v_3}{\sqrt{2}} & \frac{g' v_3}{\sqrt{2}} & 0 \end{pmatrix} \quad (8)$$

where the successive rows and columns correspond to $(\tilde{H}_2, \tilde{H}_1, -i\tilde{W}_3, -i\tilde{B}, \nu_\tau)$. Here

$$v(v') = \sqrt{2} \left(\frac{m_Z^2}{g^2} - \frac{v_3^2}{2} \right)^{1/2} \sin \beta (\cos \beta).$$

M and M' are the $SU(2)$ and $U(1)$ gaugino mass parameters respectively, μ the Higgsino mass parameter, and \bar{g}

$= \sqrt{g^2 + g'^2}$. Evidently, this will result in the generation of (Majorana) masses for the third generation neutrino at the tree level through a seesaw type mechanism [17].

In such a situation, the (3×3) chargino mass matrix is

$$M_{\tilde{\chi}^\pm} = \begin{pmatrix} M & -g v_2 & 0 \\ -g v_1 & \mu & f v_3 \\ -g v_3 & \epsilon & -f v_1 \end{pmatrix} \quad (9)$$

where $v_1 = \langle H_1 \rangle$, $v_2 = \langle H_2 \rangle$, $v_3 = \langle \tilde{\nu}_\tau \rangle$, $\tan \beta = v_2 / v_1$ and $f = h_{33}^l = m_\tau / v_1$, M being the $SU(2)$ gaugino mass parameter. Here we have assigned $(-i\tilde{W}^-, \tilde{H}_1^-, \tilde{\tau}_L^-)$ along the rows and $(-i\tilde{W}^+, \tilde{H}_2^+, \tilde{\tau}_R^+)$ along the columns.

Similarly, in the scalar sector neutral and charged scalar mass matrices are enlarged as a result of the mixing between charged sleptons and charged Higgs bosons and sneutrino with the neutral Higgs bosons. In the neutral scalar sector the scalar mass-squared matrix in the basis $\{Re(H_1), Re(H_2), Re(\tilde{\nu}_\tau)\}$ takes the form

$$M_s^2 = \begin{pmatrix} m_1^2 + 2\lambda c + 4\lambda v_1^2 & -4\lambda v_1 v_2 + B_1 \mu & 4\lambda v_1 v_3 + \mu \epsilon \\ -4\lambda v_1 v_2 + B_1 \mu & m_2^2 - 2\lambda c + 4\lambda v_2^2 & -4\lambda v_3 v_2 + B_2 \epsilon \\ 4\lambda v_1 v_3 + \mu \epsilon & -4\lambda v_3 v_2 + B_2 \epsilon & m_{\tilde{\nu}_\tau}^2 + 2\lambda c + 4\lambda v_3^2 \end{pmatrix}. \quad (10)$$

In a similar way, the neutral pseudoscalar mass-squared matrix becomes 3×3 in the basis $\{Im(H_1), Im(H_2), Im(\tilde{\nu}_\tau)\}$ and takes the form

$$M_p^2 = \begin{pmatrix} m_1^2 + 2\lambda c & -B_1 \mu & \mu \epsilon \\ -B_1 \mu & m_2^2 - 2\lambda c & -B_2 \epsilon \\ \mu \epsilon & -B_2 \epsilon & m_{\tilde{\nu}_\tau}^2 + 2\lambda c \end{pmatrix}. \quad (11)$$

In the charged scalar sector the charged scalar mass-squared matrix becomes 4×4 in the basis $\{H_2, H_1, \tilde{\tau}_L, \tilde{\tau}_R\}$ and is given by

$$M_c^2 = \begin{pmatrix} r - \frac{1}{4} g'^2 c & -B_1 \mu + \frac{1}{2} g^2 v_1 v_2 & -B_2 \epsilon + \frac{1}{2} g^2 v_2 v_3 & -\epsilon f v_1 \\ -B_1 \mu + \frac{1}{2} g^2 v_1 v_2 & s + \frac{1}{4} g'^2 c + f^2 v_3^2 & \mu \epsilon + \frac{1}{2} g^2 v_1 v_3 - \frac{1}{2} f^2 v_1 v_3 & -\epsilon f v_2 - A f v_3 \\ -B_2 \epsilon + \frac{1}{2} g^2 v_2 v_3 & \mu \epsilon + \frac{1}{2} g^2 v_1 v_3 - \frac{1}{2} f^2 v_1 v_3 & p + \frac{1}{4} g^2 t + \frac{1}{4} g'^2 c & \mu f v_2 + A f v_1 \\ -\epsilon f v_1 & -\epsilon f v_2 - A f v_3 & \mu f v_2 + A f v_1 & q - \frac{1}{2} g'^2 c + f^2 v_3^2 \end{pmatrix} \quad (12)$$

with

$$r = m_2^2 + \frac{1}{4} g^2 (v_1^2 + v_2^2 + v_3^2)$$

$$s = m_1^2 + \frac{1}{4} g^2 (v_1^2 + v_2^2 - v_3^2)$$

$$p = m_L^2 + f^2 v_1^2$$

$$q = m_R^2 + f^2 v_1^2$$

$$t = (-v_1^2 + v_2^2 + v_3^2)$$

$$c = (v_1^2 - v_2^2 + v_3^2)$$

$$\lambda = (g^2 + g'^2)/8.$$

In addition to constraints coming from electroweak symmetry breaking, the requirement of electric charge conservation subjects the parameters in the potential to appropriate conditions in any given scenario.

In the calculations here, we make the following choice of independent parameters:

$$\{\mu, \tan \beta, \epsilon, v_3, m_{\tau_L}^2 = m_{\tau_R}^2 = m_{\nu_\tau}^2 = m_0^2, A_t, A_b, A_\tau, B_1\}.$$

We also take the remaining scalar mass parameters to be as follows:

$$m_{t_L}^2 = m_{t_R}^2 = m_{b_L}^2 = m_{b_R}^2 = m_q^2$$

and

$$m_{e,\mu}^2 = m_0^2$$

where $\tan \beta$ is the ratio of two Higgs vacuum expectation values, B_1 is the usual B term corresponding to the μ term, and A 's are the trilinear soft breaking terms. We have considered degenerate squark masses at low energy, except for top squark and bottom squark where left-right mixing has been taken into account.

The bilinear R -parity violating parameters ϵ and v_3 can be constrained from different experimental considerations such as the tau mass measured within the existing errors and, most importantly, from the constraints on neutrino masses. The latter can be subjected to current laboratory bounds, but the restriction becomes more stringent on using the SuperKamiokande results on atmospheric neutrinos. In this light, the values of ϵ and v_3 have to be constrained in such a way that the sneutrino VEV is less than a few hundred keV's in the basis where ϵ is rotated away.¹

The above kind of mixing in the chargino, neutralino and scalar sectors results in physical states which are superpositions of neutral (charged) leptons and neutralinos (charginos), on the one hand, and Higgs bosons and sleptons (sneutrinos) on the other. This implies that the Yukawa and gaugino coupling terms, written in terms of the physical states, will give rise to *all* the interactions of the λ and λ' types. In addition, the fact that gaugino couplings play a role here makes it possible to free some of the terms from constraints due to the gauge invariance of the superpotential. One such interaction term has particular importance in our calculation, namely one involving a top quark, a top squark and a state which is dominantly a neutrino. This is clearly a result of neutrino-neutralino mixing and the quark-squark neutralino interaction in the Lagrangian. For more details the reader is referred to [18].

¹Of course, bilinear terms $L_i H_2$ with $i=2,3$ have to be included in order to explain the ν_μ - ν_τ oscillation. This requires a straightforward extension of the formalism described above. For more details, the reader is referred to Ref. [17].

Also, a scenario such as this allows one to have tau-neutralino- W and neutrino-neutralino- Z couplings. This results in additional decay modes of a neutralino which are characteristic of bilinear R -parity violation [18].

$$\text{Case (2): } W_k = \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c$$

In such a case the requirement of gauge invariance implies that λ_{ijk} 's are antisymmetric in the first two indices due to $SU(2)$ invariance but λ'_{ijk} 's are not subjected to such constraints. Thus we have 9 λ and 27 λ' -type couplings other than the MSSM parameter space. These trilinear couplings can also be constrained in various ways like, for example, lepton universality violation [19], neutrinoless double beta decay [20], Majorana mass of the neutrino [21], flavor changing neutral current processes [22], etc. Their implications in various high energy collider experiments such as the CERN e^+e^- collider LEP [23], DESY ep collider HERA [24] or the Fermilab Tevatron [25] have also been widely explored. In addition, they can be responsible for neutrino masses generated at the one-loop level [26].

Before we end this section, it may be remarked that although constraints on the R -violating parameters are frequently talked about, in practice these constraints will always have to stipulate a given set of values of the R -conserving (MSSM) parameters.

III. ONE-LOOP CALCULATIONS

The one-loop calculation of the decay $\tilde{\chi}_1^0(p) \rightarrow \nu(k_1) + \gamma(k_2)$ has been performed in the nonlinear R gauge. A similar calculation for the radiative decay of heavier neutralinos into lighter ones can be found in Ref. [27]. In this gauge, the choice of the gauge fixing term modifies certain types of vertices compared to, say, the 't Hooft-Feynman gauge. For example, the $W^+ G^- \gamma$ vertex (where G^\pm is the charged Goldstone boson) is absent in the non-linear R gauge, whereas the $W^+ W^- \gamma$ vertex gets correspondingly modified. Also a set of diagrams involving a Z - γ transition (with an off-shell Z), which potentially contributes to the process under investigation, gives zero contribution when one sums over all the loops.

The diagrams which ultimately contribute in the bilinear R -parity violating scenario are the triangle diagrams shown in Fig. 1. A subset of these will be present when one considers only trilinear lepton number violating terms in the superpotential, the relevant diagrams being (a) and (b) involving only the down-type quark or squark or lepton or sleptons in the loop. This is actually a consequence of the $SU(2)$ invariance of the superpotential and can be seen explicitly if one expands the λ - and λ' -type terms into their component forms. The appearance of additional diagrams in the bilinear R -parity violating case can be attributed to mixing in the scalar and lepton-chargino-neutralino sectors, once electroweak symmetry breaking takes place (see Sec. II). In fact, as has already been mentioned, one of the main reasons for undertaking this study with bilinear R -parity violation is the presence of the additional diagrams involving the top quark and the W boson, which are potential sources of enhancement of the decay amplitude.

Since the photon is on shell, $U(1)_{EM}$ gauge invariance demands that the transition amplitude be proportional to $\sigma^{\mu\nu} k_{2\mu} \epsilon_\nu^*$. Thus one expects the following form for the matrix element:

$$\mathcal{M} = i g_{\tilde{\chi}_1^0 \nu \gamma} \bar{u}(k_1) (P_R - \eta_\nu \eta_1 P_L) \sigma^{\mu\nu} k_{2\mu} \epsilon_\nu^* u(p) \quad (13)$$

where $M_{\tilde{\chi}_1^0}$ is the mass of $\tilde{\chi}_1^0$, and η_1 and η_ν are the signs of the mass eigenvalues of $\tilde{\chi}_1^0$ and the neutrino respectively. Thus, depending on η_1 and η_ν , the effective $\tilde{\chi}_1^0 \nu \gamma$ interaction is either proportional to $\sigma^{\mu\nu} k_{2\mu} \epsilon_\nu^*$ or $\gamma_5 \sigma^{\mu\nu} k_{2\mu} \epsilon_\nu^*$. The radiative decay width of $\tilde{\chi}_1^0$ is then given by

$$\Gamma(\tilde{\chi}_1^0 \rightarrow \nu \gamma) = \frac{g_{\tilde{\chi}_1^0 \nu \gamma}^2 M_{\tilde{\chi}_1^0}^3}{16\pi} \quad (14)$$

where we have neglected the mass of the neutrino and $g_{\tilde{\chi}_1^0 \nu \gamma}^2$, containing details of the loop integrals, has the dimension of inverse mass squared.

In order to calculate $g_{\tilde{\chi}_1^0 \nu \gamma}$ one has to evaluate the triangle graphs. For a given set of internal particles there are two diagrams which differ from each other by the direction of charge flow in the loop. In the figures we have shown only one set of diagrams. One must be careful about the diagrams involving the reverse flow of charge in the loop because in those cases one encounters vertices with clashing arrows.

The integrals which appear during the loop calculations are regularized using dimensional regularization. Since there is no tree-level coupling of the type $\tilde{\chi}_1^0 \nu \gamma$ the divergences must cancel among each gauge invariant subset of diagrams. For example, diagrams (g) and (h) and their partner graphs where the direction of charge flow is reversed form a gauge invariant subset and it can be shown that the infinities cancel within this set of diagrams. Similarly, one gets a finite result out of the loop diagrams involving fermion and sfermion loops, i.e., the diagrams {(a), (b)} and their partner graphs. This is true, pairwise, for the loops {(c),(d)} and {(e),(f)} involving charged Higgs bosons and charginos and charged Goldstone bosons and charginos.

The decay amplitude shown in Eq. (13) can be decomposed in the following way (with only the appropriate contributions retained, as stated earlier, with trilinear R -violating terms):

$$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2. \quad (15)$$

The matrix element due to the four diagrams (a), (b) and their partner graphs is given by

$$\begin{aligned} \mathcal{M}_1 = & \frac{e g^2 \eta_{\nu-}}{16\pi^2} \bar{u}(k_1) (P_R - \eta_\nu \eta_1 P_L) \not{k}_2 \not{\epsilon}^* u(p) \\ & \times \sum_f e_f C_f \{ (A_L B_R - A_R B_L) [\eta_1 M_{\tilde{\chi}_1^0} (I^1 - I^3)] \\ & + m_f (A_L B_L - A_R B_R) I^2 \} \end{aligned} \quad (16)$$

where Q_f is the charge of the fermion f in units of e ($e > 0$), and C_f is the color factor for the particles in the loop. One can use this result to include contributions from all possible fermion and scalar loops. Thus contributions from charged

Higgs and Goldstone bosons are also present in the sum over f . We have also explicitly used the sum over the three possible chargino states ($\tilde{\chi}_k^\pm$) in the bilinear R -parity violating scenario. One must be careful with the masses and couplings as well as with the electric charges of the fermions in the loop. The integrals I^i are expressible in terms of the general expressions for one-loop three-point functions [28], in forms that are presented in the Appendix, where we also give the expressions for the combinations $(A_L B_L - A_R B_R)$ and $(A_L B_R - A_R B_L)$ for all possible cases.

In the case of the top squark and bottom squark we have included the effect of left-right mixing. Also, the calculation includes possible absorptive parts of the loop integrals, which can be present in particular when the decaying neutralino is heavier than the W .

In a similar way, the contributions to the matrix element from diagrams (g) and (h) and their partner graphs are given by

$$\begin{aligned} \mathcal{M}_2 = & \frac{-e g^2 \eta_{\nu-}}{4\pi^2} \bar{u}(k_1) (P_R - \eta_\nu \eta_1 P_L) \not{k}_2 \not{\epsilon}^* u(p) \\ & \times \sum_k \{ (A_L B_L - A_R B_R) [\eta_1 M_{\tilde{\chi}_1^0} (I^1 - J^2 - I^3)] \\ & + 2M_k (A_L B_R - A_R B_L) J^2 \}. \end{aligned} \quad (17)$$

In both of the above expressions, the quantities A_L , A_R , B_L and B_R are all assumed to be real.

The loops involving the top quark [Figs. 1(a) and 1(b)], which are present only in the case of bilinear R -parity violation, always seem to have a rather important effect.² Matching contributions also come over a large area of the parameter space from loops involving the W boson (and the tau and/or the lighter chargino, depending on the relevant mixing angles) in the propagator [diagrams (g) and (h)]. The very presence of these diagrams causes the rates for radiative decays to be larger in cases with bilinear R -violating effects than in those with only trilinears, except in those where there is destructive interference between the two types of graphs.

IV. NUMERICAL RESULTS

The calculation of the branching ratio for $\tilde{\chi}_1^0 \rightarrow \nu \gamma$ involves the computation of decay widths of the lightest neutralino in all the relevant tree-level two- and three-body decay modes. The different types of mixing that have been discussed in Sec. II for bilinear R -parity violation open up the following two-body decay modes, as and when kinematically allowed:

²Our detailed scan of the parameter space, however, also reveals regions where lighter fermions contribute comparably because of favored mixing angles, much in the same way as the charm quark makes dominant contributions to the real part of the box diagrams for $K^0 - \bar{K}^0$ mixing.

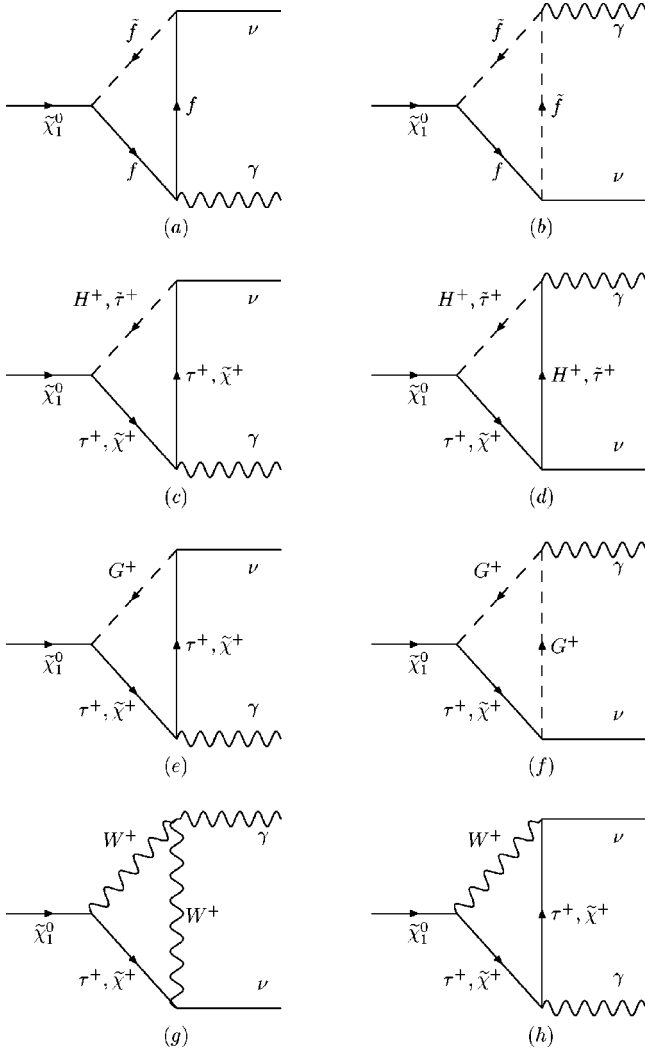


FIG. 1. One-loop contributions to the decay $\tilde{\chi}_1^0 \rightarrow \nu \gamma$. In addition, for every diagram there is a counterpart with the internal fermion line(s) reversed.

$$\tilde{\chi}_1^0 \rightarrow \tau^\pm W^\pm, \tilde{\chi}_1^0 \rightarrow \nu_\tau Z, \tilde{\chi}_1^0 \rightarrow \nu_\tau h$$

where h is the lightest Higgs boson.

In addition, there are three body tree-level decay channels with both bilinears and trilinears. While the channels $\tilde{\chi}_1^0 \rightarrow e_i^+ \bar{u}_j d_k$, $\tilde{\chi}_1^0 \rightarrow \nu_i \bar{d}_j d_k$, $\tilde{\chi}_1^0 \rightarrow e_i^+ \nu_j e_k^-$ are available in both cases [29], additional decays such as $\tilde{\chi}_1^0 \rightarrow u_i \bar{u}_i \nu$ and $\tilde{\chi}_1^0 \rightarrow \nu \nu \nu$ can take place in the former, and can be important when two-body decays are not kinematically allowed.

In all the numerical results presented here, we have used $A_t = A_b = 10$ GeV. Our finding is that the branching ratio of the radiative decay is rather insensitive to the values of these parameters.

Let us first concentrate on the bilinear R -parity violating scenario. We will consider two different cases, namely (i) when the supersymmetry breaking gaugino mass parameters are unified at the grand unification scale and (ii) when $SU(2)$ and $U(1)$ gaugino mass parameters denoted by M_2 and M_1 respectively are treated as unrelated.

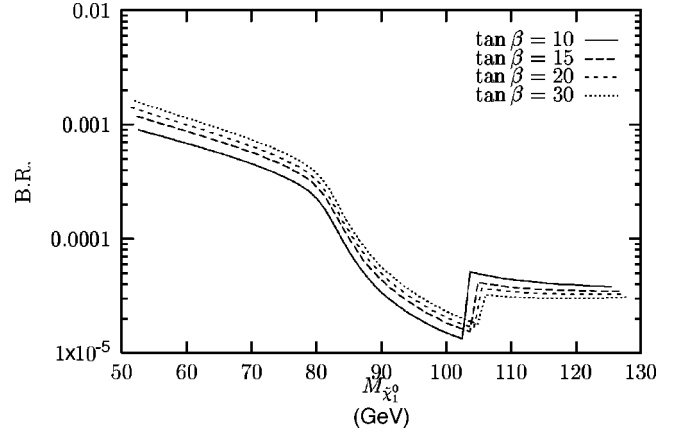


FIG. 2. Branching ratio for the decay $\tilde{\chi}_1^0 \rightarrow \nu \gamma$ (with SK). The remaining supersymmetric parameters are chosen as $\mu = 200$ GeV, $B_1 = -200$ GeV, $m_{\tilde{l}} = m_{\tilde{\nu}} = 200$ GeV, and $m_{\tilde{g}} = 600$ GeV.

For reasons explained below, significant contributions to the radiative decay can come only when bilinear R -parity violating effects are present. There, again, the relevant parameters ϵ and the sneutrino VEV(s) can be constrained under two different considerations. If the neutrino-neutralino mixing process in such a scenario is the main mechanism for the generation of (Majorana) neutrino masses for explaining the SuperKamiokande (SK) results on atmospheric muon neutrinos, then the parameters have to be constrained in such a way that the quantity $v' = \sqrt{v_\mu^2 + v_\tau^2}$ is less than about 100 keV in a basis where the ϵ parameters are rotated away from the superpotential. The numerical results modulo such constraints are denoted by “with SK” in the corresponding figure captions, and the results correspond to both the second- and third-generation neutrinos in the final state of the radiative decays. On the other hand, if one ignores the SK constraints, then, with just the laboratory bounds on the tau neutrino mass, the values of ϵ and v_3 (i.e. the R -violating parameters in the third generation) can be as large as on the order of GeV’s. We have also presented such results, considering only the third (tau) neutrino to be there as the decay product.

Figures 2–9 (10) show the results of a scan of the parameter space carried out in the different scenarios mentioned above, calculated with bilinear (trilinear) R -parity violation. The first thing that we note is that in all the results presented, a specific set of masses for the squarks and sleptons is assumed. The branching ratios of the radiative decay, however, are rather insensitive to their variation. This is because if, for example, we reduce the squark masses, the effect of a lighter top squark will increase the decay width for the radiative decay, but the corresponding tree level decays will also undergo a boost via diagrams mediated by squarks.

Similarly, a comparison of Figs. 2–4 with 5 and 6, and Figs. 7 and 8 with 9 reveals that imposing the SK constraints do not cause any appreciable reduction to the probability of obtaining branching ratios on the higher side for similar combinations of MSSM parameters. All that it does is to scale the overall coefficients instrumental in both the loop-

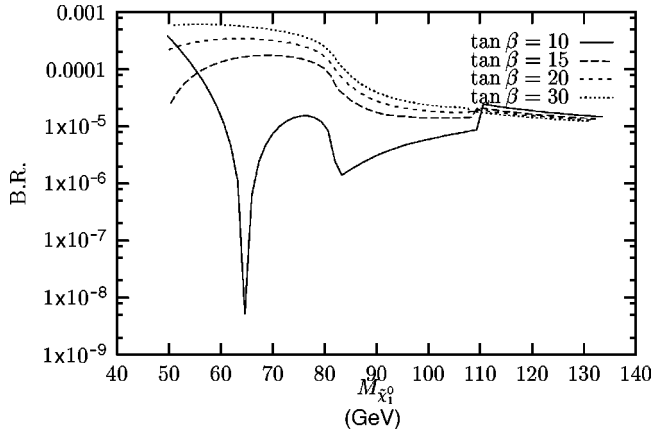


FIG. 3. Branching ratio for the decay $\tilde{\chi}_1^0 \rightarrow \nu \gamma$ (with SK). The remaining supersymmetric parameters are chosen as $\mu = -200$ GeV, $B_1 = 200$ GeV, $m_{\tilde{l}} = m_{\tilde{\nu}} = 200$ GeV, and $m_{\tilde{q}} = 600$ GeV.

and tree-level decays. This results in branching ratios of similar magnitudes, but is manifested in larger decay lengths for the neutralino when R -parity violating parameters are restricted to yield $\Delta m_{23}^2 \sim \text{a few times } 10^{-3} \text{ eV}^2$.

As has already been mentioned, the major contributions to the radiative decay come from (a) the top-quark–top-squark loop with bilinear R -parity violation, as well as from loops involving the W boson and the tau or the lighter chargino. However, this leads in some regions of the parameter space to the interesting possibility of their cancelling each other. When such a cancellation is very severe for some specific neutralino mass, the branching ratio is seen to undergo a sharp dip at that point, as seen in Figs. 3 and 8. In such cases, the effective contributions at those points hardly contain any input that is special to the bilinear terms. On the other hand, in Fig. 2 (and partially in Fig. 3) one encounters a situation where the contribution is mainly from the W loops but the latter undergoes a destructive interference among the various component terms, causing the overall branching ratio to fall at a particular region.

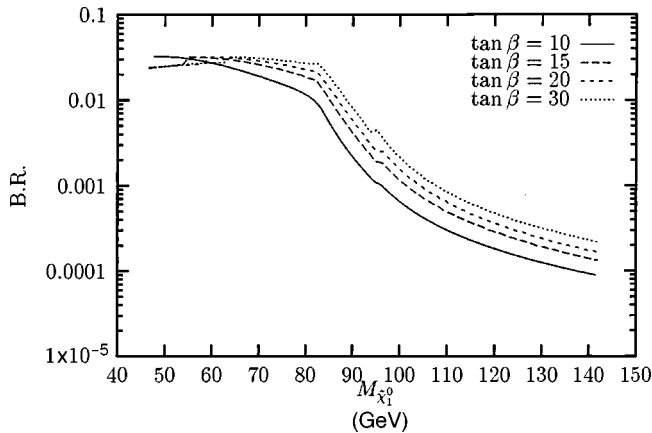


FIG. 4. Branching ratio for the decay $\tilde{\chi}_1^0 \rightarrow \nu \gamma$ (with SK). The remaining supersymmetric parameters are chosen as $\mu = 500$ GeV, $B_1 = -200$ GeV, $m_{\tilde{l}} = m_{\tilde{\nu}} = 200$ GeV, and $m_{\tilde{q}} = 600$ GeV.

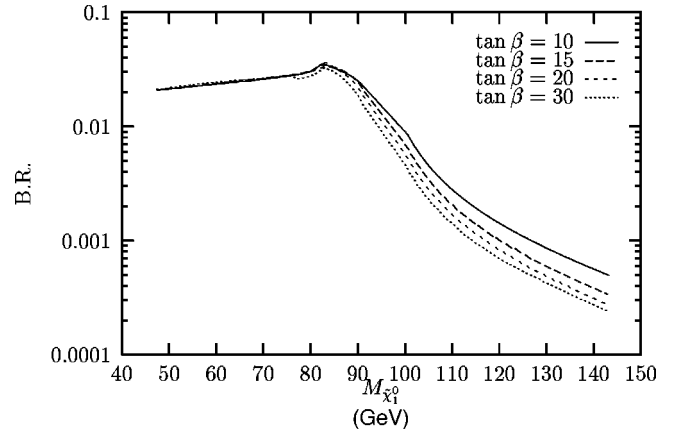


FIG. 5. Branching ratio for the decay $\tilde{\chi}_1^0 \rightarrow \nu \gamma$. The remaining supersymmetric parameters are chosen as $\mu = -500$ GeV, $\epsilon = 10$ GeV, $v_3 = 1$ GeV, $B_1 = 200$ GeV, $m_{\tilde{l}} = m_{\tilde{\nu}} = 200$ GeV, and $m_{\tilde{q}} = 600$ GeV.

The numerical results show a sensitivity to the MSSM parameters μ and, to a somewhat lesser extent, to $\tan \beta$. It is also clear from the graphs that the radiative decay tends to remain suppressed for small values of $|\mu|$ which is treated here as a free parameter. It gradually rises with $|\mu|$, and then almost saturates, showing a slight fall for $|\mu|$ approaching 1 TeV. The loops have the largest contributions for $|\mu| \sim 500$ GeV. By and large, the corresponding regions of the parameter space have the lightest neutralino almost entirely dominated by the B -ino state.

When the condition of gaugino mass unification is relaxed, the mass parameters M_1 and M_2 , corresponding to the $U(1)$ and $SU(2)$ gauginos can be unrelated [30]. Most of the features of the unified scenario, including the possibilities of having branching ratios close to 10%, are seen here also. However, it entails the additional possibility of having a destructive interference between the top-quark- and W -induced diagrams in cases where the branching ratio is otherwise on the higher side, causing the latter to fall sharply by about 5 orders of magnitudes for a particular value of the neutralino

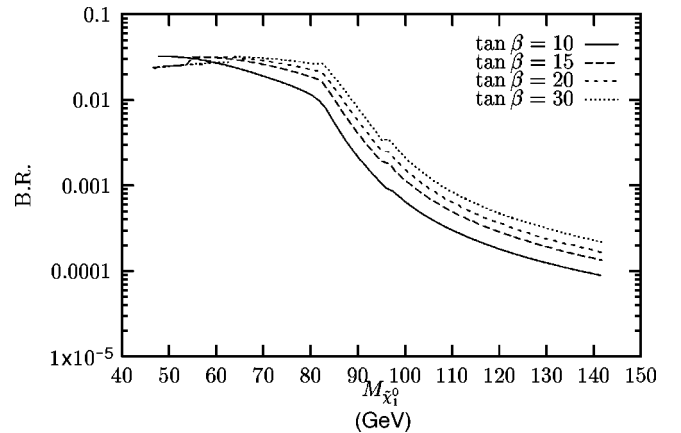


FIG. 6. Branching ratio for the decay $\tilde{\chi}_1^0 \rightarrow \nu \gamma$. The remaining supersymmetric parameters are chosen as $\mu = 500$ GeV, $\epsilon = 10$ GeV, $v_3 = 1$ GeV, $B_1 = -200$ GeV, $m_{\tilde{l}} = m_{\tilde{\nu}} = 200$ GeV, and $m_{\tilde{q}} = 600$ GeV.

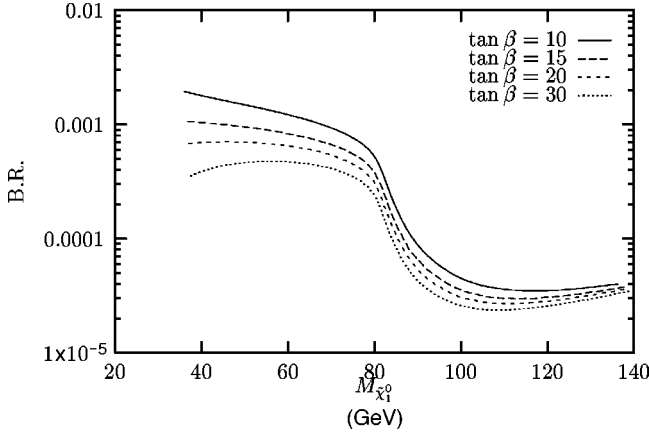


FIG. 7. Branching ratio for the decay $\tilde{\chi}_1^0 \rightarrow \nu \gamma$ (with SK), where M_1 and M_2 are free parameters. The value of M_2 is taken to be 300 GeV. The remaining supersymmetric parameters are chosen as $\mu = 200$ GeV, $B_1 = -200$ GeV, $m_{\tilde{l}} = m_{\tilde{\nu}} = 200$ GeV, and $m_{\tilde{q}} = 600$ GeV.

mass, with all other parameters at the same values. This is seen when M_2/M_1 is smaller than what it would have been with the constraint of unification, and the dip is found to occur when the two masses are quite close to each other.

Figure 10 is a sample showing the order of magnitude of the radiative decay with only trilinear R -violating couplings present in the theory. The values of the λ - and λ' -type coupling constants have been used consistently with the current limits [31]. We have already seen that the two potentially most important classes of diagrams, namely, those mediated by the top quark and the W , are absent in such a case. Therefore, it is hardly surprising that the branching ratios cannot rise above 10^{-8} – 10^{-10} over a large part of the parameter space. The range in which the branching ratio tends to lie in such a case matches, as one would expect, the one to which it falls when a cancellation of the large contributions takes place in the scenario with bilinears. In any case, the radiative branching ratio turns out to be too small to be of any observ-

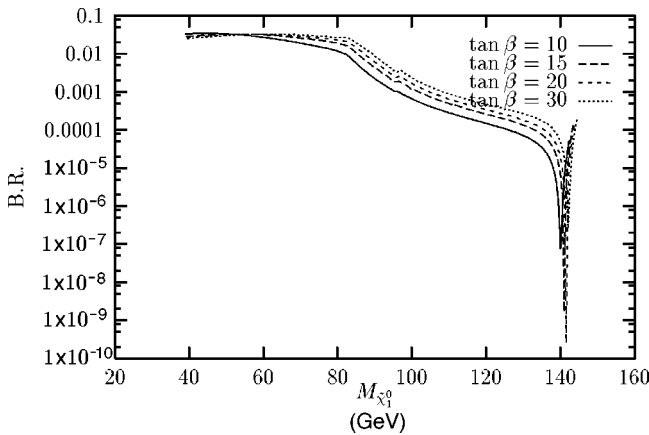


FIG. 8. Branching ratio for the decay $\tilde{\chi}_1^0 \rightarrow \nu \gamma$ (with SK), where M_1 and M_2 are free parameters. The value of M_2 is taken to be 150 GeV. The remaining supersymmetric parameters are chosen as $\mu = 500$ GeV, $B_1 = -200$ GeV, $m_{\tilde{l}} = m_{\tilde{\nu}} = 200$ GeV, and $m_{\tilde{q}} = 600$ GeV.

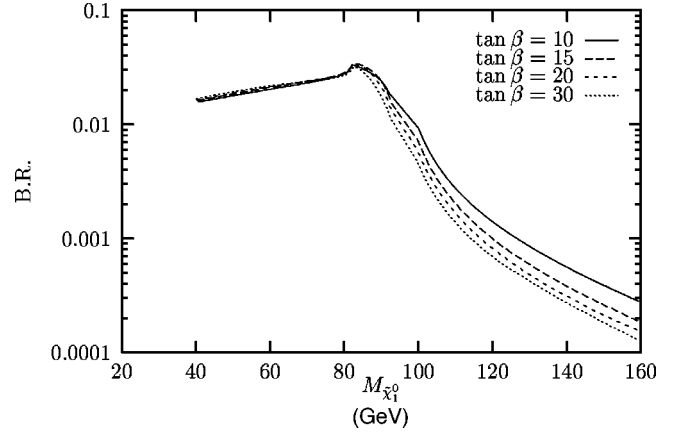


FIG. 9. Branching ratio for the decay $\tilde{\chi}_1^0 \rightarrow \nu \gamma$, where M_1 and M_2 are free parameters. The value of M_2 is taken to be 300 GeV. The remaining supersymmetric parameters are chosen as $\mu = -500$ GeV, $\epsilon = 10$ GeV, $v_3 = 1$ GeV, $B_1 = 200$ GeV, $m_{\tilde{l}} = m_{\tilde{\nu}} = 200$ GeV, and $m_{\tilde{q}} = 600$ GeV.

able significance where R parity is violated only through trilinear terms.

With the maximum value of the branching ratio for the radiative decay being between 5% and 10%, two-photon signals from such decays will fake signals like those of GMSB in only a rather small region of the parameter space. Such a thing might happen when the lightest neutralino in GMSB is close to the kinematic limit of production. However, the fact that only the bilinear R -violating terms can boost the branching ratio to the level of close to 10% has rather interesting implications in terms of observing distinctive signals of the latter. For example, pair-produced neutralinos at LEP energies (assuming an integrated luminosity of 500 pb^{-1}) can give rise to about 40 events where there is a radiative decay on one side, leading to signals of the type $3f + \gamma$. There can be many more such events in a high-energy electron-positron collider with an integrated luminosity of about 50 fb^{-1} . With a proper event selection strategy, radiative neutralino decay can thus be an interesting signature of R -parity violation with bilinear terms.

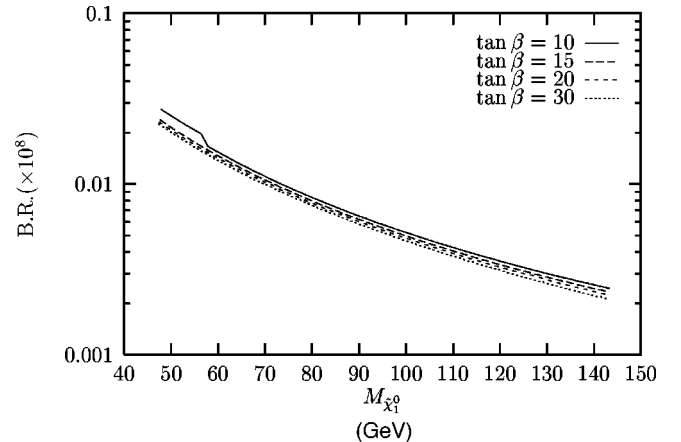


FIG. 10. Branching ratio for the decay $\tilde{\chi}_1^0 \rightarrow \nu \gamma$ in the scenario where only λ and λ' terms are present. The remaining supersymmetric parameters are chosen as $\mu = -500$ GeV, $m_{\tilde{l}} = 200$ GeV, and $m_{\tilde{q}} = 600$ GeV.

V. SUMMARY AND CONCLUSIONS

We have performed a detailed calculation of the branching ratios for the radiative decay $\tilde{\chi}_1^0 \rightarrow \nu \gamma$ for the lightest neutralino in R -parity violating SUSY models. It is seen that the branching ratio can have a maximum value of about 5%–10% when R -parity violation has its origin in the so-called “bilinear” terms in the superpotential, and is insignificantly smaller for that induced by “trilinear” terms. This, we point out, can be an interesting way of obtaining characteristic signals for the former type of scenario. Our conclusion is that the chances of such radiative decays faking the two-photon signals for gauge-mediated SUSY breaking are small, except where the lightest neutralino in the latter is close to the kinematic limit of production. It is also seen that the accessible region corresponding to the MSSM parameter space does not change appreciably upon subjecting the theory to constraints from neutrino mass patterns required by the SuperKamio-
kande data on atmospheric neutrinos.

ACKNOWLEDGMENTS

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APPENDIX

Here we outline the actual forms of the various quantities used in the loop calculations in Sec. III.

We give expressions for the mixing parameters A_L , B_L , A_R , B_R for W^\pm loops in the bilinear R -parity violating scenario:

$$A_L = \frac{1}{\sqrt{2}} Z_{i1} V_{k2} - Z_{i3} V_{k1} \quad (A1)$$

$$B_L = \frac{1}{\sqrt{2}} Z_{j1} V_{k2} - Z_{j3} V_{k1} \quad (A2)$$

$$A_R = -\frac{1}{\sqrt{2}} Z_{i2} U_{k2} - Z_{i3} U_{k1} + \frac{1}{\sqrt{2}} Z_{i5} U_{k3} \quad (A3)$$

$$B_R = -\frac{1}{\sqrt{2}} Z_{j2} U_{k2} - Z_{j3} U_{k1} + \frac{1}{\sqrt{2}} Z_{j5} U_{k3} \quad (A4)$$

where we have $j=2$ and $i=1$. Similarly for charged scalar loops we have

$$A_L = m' (Z_{i5} V_{k3} C_{2m} - Z_{i2} V_{k3} C_{3m}) + 2Q_\tau \tan \theta_W Z_{i4} V_{k3} C_{4m} + \sqrt{2} \left\{ Z_{i1} V_{k1} + \frac{1}{\sqrt{2}} (Z_{i4} V_{k2} \tan \theta_W + Z_{i3} V_{k3}) \right\} C_{1m} \quad (A5)$$

$$A_R = -m' Z_{i2} U_{k3} C_{4m} + (Z_{i3} - Z_{i4} \tan \theta_W (1 + 2Q_\tau)) U_{k3} C_{3m} + \sqrt{2} \left\{ Z_{i2} U_{k1} - \frac{1}{\sqrt{2}} (Z_{i4} U_{k2} \tan \theta_W + Z_{i3} U_{k3}) \right\} C_{2m} \quad (A6)$$

$$B_L = -m' Z_{j2} U_{k3} C_{4m} + (Z_{j3} - Z_{j4} \tan \theta_W (1 + 2Q_\tau)) U_{k3} C_{3m} + \sqrt{2} \left\{ Z_{j2} U_{k1} - \frac{1}{\sqrt{2}} (Z_{j4} U_{k2} \tan \theta_W + Z_{j3} U_{k3}) \right\} C_{2m} \quad (A7)$$

$$B_R = m' (Z_{j5} V_{k3} C_{2m} - Z_{j2} V_{k3} C_{3m}) + 2Q_\tau \tan \theta_W Z_{j4} V_{k3} C_{4m} + \sqrt{2} \left\{ Z_{j1} V_{k1} + \frac{1}{\sqrt{2}} (Z_{j4} V_{k2} \tan \theta_W + Z_{j3} V_{k3}) \right\} C_{1m} \quad (A8)$$

where

$$m' = \frac{m_\tau}{(m_W^2 - g^2 v_3^2/2)^{1/2} \cos \beta}. \quad (A9)$$

Z 's are the neutralino mixing elements, U 's and V 's are the chargino mixing elements and C 's are the charged scalar mixing elements.

There are special cases for the top quark and bottom squark: we have considered the effect of mixing between \tilde{t}_L and \tilde{t}_R as well as between \tilde{b}_L and \tilde{b}_R . (i) For the t - \tilde{t}_1 triangle,

$$A_L = (Z_1^- + 2Q_t Z_{14} \tan \theta_W) (-\sin \theta_t) + \frac{1}{m_W \sin \beta} m_t Z_{11} \cos \theta_t \quad (A10)$$

$$A_R = \frac{m_t}{m_W \sin \beta} Z_{11} (-\sin \theta_t) - 2Q_t \tan \theta_W Z_{14} \cos \theta_t \quad (A11)$$

$$B_L = \frac{m_t}{m_W \sin \beta} Z_{21} (-\sin \theta_t) - 2Q_t \tan \theta_W Z_{24} \cos \theta_t \quad (A12)$$

$$B_R = (Z_2^- + 2Q_t Z_{24} \tan \theta_W) (-\sin \theta_t) + \frac{1}{m_W \sin \beta} m_t Z_{21} \cos \theta_t. \quad (A13)$$

In our convention, \tilde{t}_1 is the lightest physical top squark and couplings or \tilde{t}_2 can be obtained by replacing $-\sin \theta_t \rightarrow \cos \theta_t$ and $\cos \theta_t \rightarrow \sin \theta_t$

(ii) For b - \tilde{b}_1 triangle,

$$A_L = (-Z_1^- + 2Q_b Z_{14} \tan \theta_W) (-\sin \theta_b) + \frac{1}{m_W \sin \beta} m_b Z_{12} \cos \theta_b \quad (A14)$$

$$A_R = \frac{m_b}{m_W \sin \beta} Z_{12} (-\sin \theta_b) - 2Q_b \tan \theta_W Z_{14} \cos \theta_b \quad (\text{A15})$$

$$B_L = \frac{m_b}{m_W \sin \beta} Z_{22} (-\sin \theta_b) - 2Q_b \tan \theta_W Z_{24} \cos \theta_b \quad (\text{A16})$$

$$B_R = (-Z_2^- + 2Q_b Z_{24} \tan \theta_W) (-\sin \theta_b) + \frac{1}{m_W \sin \beta} m_b Z_{22} \cos \theta_b \quad (\text{A17})$$

with

$$Z_1^- = Z_{13} - Z_{14} \tan \theta_W \quad (\text{A18})$$

$$Z_2^- = Z_{23} - Z_{24} \tan \theta_W. \quad (\text{A19})$$

In our convention, \tilde{b}_1 is the lightest physical bottom squark and couplings or \tilde{b}_2 can be obtained by replacing $-\sin \theta_b \rightarrow \cos \theta_b$ and $\cos \theta_b \rightarrow \sin \theta_b$.

(iii) Degenerate sfermion loops: For up-type squarks we can use the same expression as that for the top squark loop without the mixing angle corresponding to $\tilde{t}_L - \tilde{t}_R$ mixing. Similarly for down-type squark or charged sleptons, one can use the expressions corresponding to the bottom squark loop modulo the mixing factor. Also the charges and the color factors should be properly included.

In the case of λ -type couplings the expressions become (for the down-type squarks)

$$A_L = 0(\lambda') \quad (\text{A20})$$

$$A_R = \lambda(0) \quad (\text{A21})$$

$$B_L = \frac{m_d}{m_W \cos \beta} Z_{12} - 2Q_d Z_{14} \tan \theta_W \quad (\text{A22})$$

$$B_R = \frac{m_d}{m_W \cos \beta} Z_{12} + 2Q_d Z_{14} \tan \theta_W - Z_1^- \quad (\text{A23})$$

where appropriate generational indices should be considered for the λ - and λ' -type couplings.

In the case of charged sleptons one should use the appropriate masses and charge. One must remember that in the case of the trilinear R -parity violating scenario we have diagonalized a 4×4 neutralino mass matrix.

The integrals corresponding to the three-point functions are

$$C_0 = \frac{1}{i\pi^2} \int d^4 q \frac{1}{[(q^2 + m_f^2)(q-p)^2 + m_s^2][(q-k_2)^2 + m_f^2]} \quad (\text{A24})$$

$$C_\mu = \frac{1}{i\pi^2} \int d^4 q \frac{q_\mu}{[(q^2 + m_f^2)(q-p)^2 + m_s^2][(q-k_2)^2 + m_f^2]} \quad (\text{A25})$$

$$C_{\mu\nu} = \frac{1}{i\pi^2} \int d^4 q \frac{q_\mu q_\nu}{[(q^2 + m_f^2)(q-p)^2 + m_s^2][(q-k_2)^2 + m_f^2]}. \quad (\text{A26})$$

They are expressed in terms of various form factors as follows:

$$C_\mu = -p_\mu C_{11} + k_{1\mu} C_{12} \quad (\text{A27})$$

$$C_{\mu\nu} = p_\mu p_\nu C_{21} + k_{1\mu} k_{1\nu} C_{22} - (p_\mu k_{1\nu} + p_\nu k_{1\mu}) C_{23} + \delta_{\mu\nu} C_{24} \quad (\text{A28})$$

and

$$k_{1\mu} = p_\mu - k_{2\mu} \quad (\text{A29})$$

$$I^1 = C_{12} - C_{11} \quad (\text{A30})$$

$$I^2 = C_0 \quad (\text{A31})$$

$$I^3 = I^4 + I'^4 \quad (\text{A32})$$

$$I^4 = C_{23} - C_{22} \quad (\text{A33})$$

and I'^4 can be obtained from I^4 by $m_s \leftrightarrow m_f$ interchange. For more details on the three-point form factors, see Ref. [28].

J^2 is analogous to I^2 but the denominator in Eq. (A24) will be changed to

$$(q^2 + m_f^2)\{(q-k_1)^2 + m_s^2\}\{(q-p)^2 + m_s^2\}.$$

The integrals which appear in the calculations of \mathcal{M}_2 are identical in form to those mentioned above but one should replace m_s with m_W and m_f with M_k . Here m_s and m_f are the masses of the scalar and the fermion appearing in the loop respectively and M_k is the mass of the chargino.

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